

Track-Guided Radar for Rapid Transit Headway Control

A. D. McAulay*

The Boeing Co., Seattle, Washington

A guided radar system is outlined which is suitable for collision avoidance and headway control in future high capacity systems, and a method is proposed in which the track is adapted for use as a radar waveguide. Present high-frequency methods are reviewed for comparison. A numerical procedure involving a generalization of the finite element method using the method of weighted residuals is developed for determining the complex propagation constants for surface waves along lossy waveguides of arbitrary shape such as the track waveguide. A strip of polystyrene attached to a rail was analyzed as a waveguide, and the results indicated that a 1.3 Ghz wave would travel about 155 m before the attenuation had dropped to 1/e of the initial signal, even in wet track conditions. The phase constant was shown to be linear over several hundred khz.

I. Introduction

THERE has been a renewed interest in tracked surface transportation systems as a result of energy and pollution problems. In certain situations tracked systems have advantages over nontracked systems,¹ in particular they are easier to automate. Automation is aimed primarily at reducing operating costs and improving ride quality and has been proceeding gradually for tracked systems.² A critical part of the automation process is the vehicle detection and communication because of the problem of safety, reliability, and cost.³ The block method for vehicle detection is described (Sec. II.). The advantages of using high frequencies and surface electromagnetic waves are discussed and current high-frequency systems under investigation are reviewed (Sec. III.). A guided radar system is outlined which is suitable for collision avoidance and headway control in a high capacity system (Sec. IV.). An innovative method is proposed in which the track is adapted for use as the radar waveguide, and an adapted personal rapid transit guideway (PRT) and a rail are presented as examples (Sec. IVB.). To analyze the track waveguide, the finite element method is generalized by means of the method of weighted residuals to develop a method of determining the complex propagation constants for surface waves along lossy waveguides of arbitrary shape (Sec. V.). The results of analyzing a railroad rail as a surface waveguide are presented and indicate the feasibility of using the track as the waveguide for a guided radar system (Sec. VI.).

II. The Block System

The fixed block system has been used since 1870⁴ and is still a basic element of all the tracked surface transportation systems operating in North America today, including the recently built systems in Toronto, Montreal, Philadelphia (Lindenwold Line), San Francisco (BART), and Morgantown (PRT).

A track is subdivided into blocks up to 1000 m in length. The presence of a vehicle in a block is detected and used to control signal lights or speed signals sent to preceding blocks. Typically a zero speed or stop signal is sent to the block immediately preceding an occupied block, and a slow signal is sent to the next preceding block.

In the case of a railroad, the vehicle is detected by coupling audio frequency electromagnetic waves into one short circuited end of a section of rail and detecting them at the other

short circuited end of the block. A train provides a short circuit between the rails and prevents the signal from reaching the far end of the block.

In the case of a rubber tired vehicle system, a block is generated by laying a loop of wire along the track to represent a block. Removal of power in this loop is interpreted on the vehicle as a requirement to apply emergency brakes. The presence of a vehicle in a block is detected by having a permanently transmitting antenna on the vehicle which is detected by a wayside antenna, a receiving loop running the length of the block. The vehicle is no longer detectable if power on the vehicle fails, thus creating a safety hazard. An alternative, used on the Morgantown PRT System, is the use of a permanent magnet on the vehicle which trips reed switches on entry to and exit from a block. A further alternative is the use of metal detectors, similar to those used at traffic lights, to indicate block entry and departure. A safety hazard may occur if the check-in/check-out sensors are falsely activated, or the block occupancy memory is disturbed.

A major disadvantage of using blocks for modern transportation systems is that the cost increases rapidly as the capacity of the system is increased. The subway systems in large cities such as New York, London, and Tokyo reach their maximum capacity during rush hours. The train lengths have been extended up to the station lengths and further capacity is possible only by building new tracks or running at closer headways than before. Personal rapid transit systems use small vehicles which carry up to ten persons and travel on demand. In this case high capacities are possible only if closer headways are used than those economically attainable with fixed block systems.

A moving block system is an alternative which enables closer headways to be achieved than with a fixed block. The blocks are not built into the track; instead a vehicle maintains a safe distance behind the vehicle in front. A suitable distance is that required to stop the vehicle based on its present speed. Moving block systems proposed by General Electric, Brown Boveri, and Siemens are described in Sec. III. A track guided radar system proposed by the author is described in Sec. IV. The fixed and moving block systems, the use of crossover wires and the choice of signalling frequencies are described in more detail in Ref. 2.

III. High Frequency Approaches

Future systems aimed at improved performance and passenger facilities tend to require higher frequencies for the corresponding aspects of control and communication. High speed systems need to transmit and receive information more frequently and more rapidly. Small headways, as in a moving block system, necessitate the faster detection and dissemination of vehicle locations. Higher data rates are required for systems involving a large number of vehicles. Telephones, televisions, and surveillance cameras may be

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*Senior Specialist Engineer.

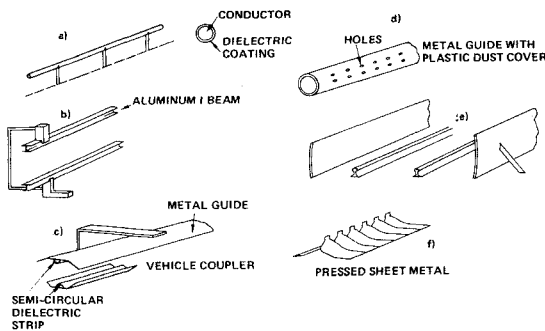


Fig. 1 High-frequency method.

desirable for passenger convenience and the reduction of crime. Moreover these requirements may occur simultaneously. The use of the highest frequency economically possible is advocated by Hu, et al.⁵

In an effort to increase the vehicular capacity by utilizing moving blocks, systems have been built by Siemens and Allgemeine Electricitts Gesellschaft and tested on the Hamburg underground in Germany⁶ and have been proposed by Brown Boveri,⁷ in which a cable transmitting frequencies of approximately 70 kHz is used to communicate continuously with the train. Vehicle separation is maintained by ensuring that the exact location of each vehicle is known at all times and this is achieved using a wire having crossovers 30 m apart throughout the track. The speed information and data messages are combined, and two way digital communication takes place about every two sec. The data message contains a train address and is transmitted using a code that will correct up to four erroneous bits per message. Higher frequencies are not useful with this cable configuration because the attenuation becomes too large above this frequency.⁷

Continuous communication with a vehicle over long distances is practical using leaky coaxial cables for the medium frequency (MF 1 MHz) and high-frequency ranges (HF 10 MHz) and wave guides for the ultra-high-frequency range (UHF 1 GHz). According to Hu et al.⁵ the cost of equipment increases only slowly up to 1 GHz. Line of sight microwave equipment is not practical most of the time because of bends, cuttings, and tunnels. The ultra high frequencies are proposed for two separate functions, one as a main continuous link with the vehicle supplying all the communication requirements, and the other as a radar for the avoidance of collisions and the detection of obstacles. The latter requirement is less stringent in terms of the allowed attenuation per kilom along the track because signals are not required to travel so far.

Various microwave systems that have been tested in conjunction with the railroads are shown in Fig. 1. A review article by Nakahara and Kurauchi⁸ provides details of the results obtained with several of these systems. The waveguide produces an external field through which the vehicle antennas pass. This field can be produced by causing a controlled radiation of energy away from the guide, or by using a surface wave in which the energy, although travelling along the guide, is partly external to the guide. This external field decays rapidly with transverse distance away from the guide and has the advantage that very little energy is lost as compared to the case where energy is continuously radiated from the guide.

The simplest practical surface waveguide is a conductor coated with a dielectric material; the so-called G line. It is fully described by Goubau.⁹ Figure 1a shows such a waveguide as tested by British Railways in 1963.¹⁰ Similar lines have been tested in the USA.¹¹ The supports must not disrupt the field and consequently nylon suspension threads and Fiberglass supports are typical. The circular symmetry of the field is a disadvantage in this application, it being preferable to have the field only on the side of the guide adjacent to the vehicle. The system is then less sensitive to adjacent obstacles and the method of mounting does not affect

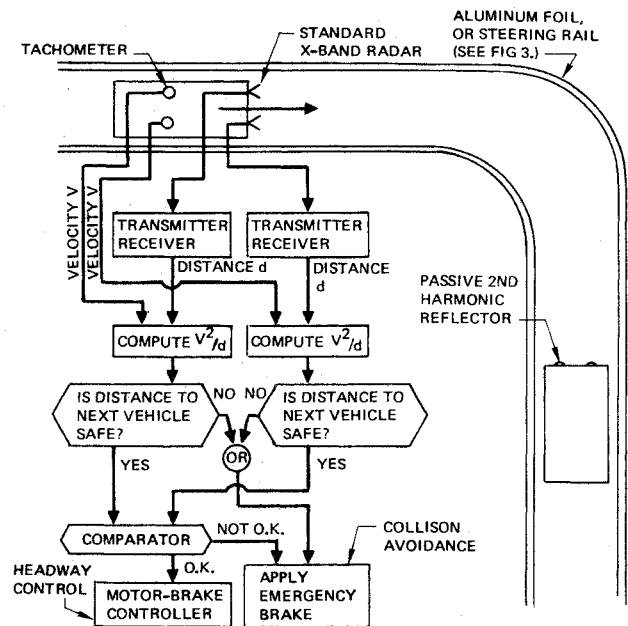


Fig. 2 Block diagram of proposed system.

the performance. Several of the other waveguide designs are aimed at achieving this asymmetry.

Figure 1b shows the transmission line system proposed by General Electric for BART in 1964.¹² A 31-Mhz carrier with an 8-khz FM signal is propagated along aluminum conductors 9 in. apart. The system detects obstacles in the path of the transmission line providing a signal to stop the train. This arrangement constitutes a moving block. All the electronics involved is on the vehicle itself, and an advantage of this system is that each train determines its own separation from the vehicle in front. Short circuits are placed across the waveguide after each station to provide signals for stopping the train correctly, and these short circuits are switched out before the train departs.

Figure 1c shows the structure proposed and tested in 1969 for transmitting information from on board television cameras back to a station for the automatic Skybus system planned in Pittsburgh. This surveillance is thought to be required to protect passengers in the case where there is no operator on the vehicle. The system, which could also be used for collision avoidance purposes was tested by General Applied Sciences for the Dept. of Transportation.¹³ The metal reflector images the electromagnetic field so that the semi-circular piece of dielectric material acts as a circular dielectric rod waveguide with permissible modes which are symmetric about a horizontal axis. The hybrid HE_{11} mode is used and the tests included the design of a repeater for installation into the waveguide without interruption of the signal.¹⁴

Figure 1d shows a leaky waveguide in which the field is permitted to leak from the waveguide through holes. Extensive experiments involving television and data transmission were conducted in Japan in 1964 on such a waveguide.¹⁵

Figure 1e shows a parallel pair of concave reflectors used to form a beam waveguide. Provided that the reflectors are large enough for the frequency selected, a low loss is obtained and the waves follow the behavior of geometric ray optics. A frequency of 9.4 GHz was used in the obstacle detection system in Ref. 8.

Figure 1f shows an economical construction for a surface waveguide, the Y guide.⁸ The periodic structure having a period smaller than the wavelength to be transmitted increases the surface reactance and enables a good surface wave to be propagated. However, the losses are large and the method was restricted to the experimental detection of vehicles and obstacles on the Tokaido 125 km/hr line in Japan.

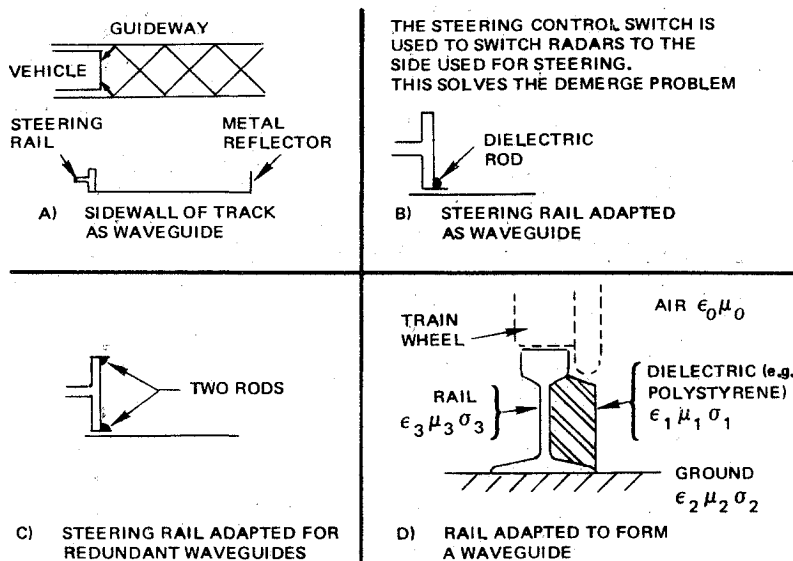


Fig. 3 Examples of track used as a waveguide.

IV. Proposed Track Guided Radar System

A. Collision Avoidance and Headway Control System

Figure 2 shows a proposed block diagram for the collision avoidance and headway control of a high capacity transportation system. The distance to the vehicle in front is measured using radar techniques. A passive reflector is used on the back of a vehicle to ensure that vehicles are detected even with loss of power. The measured distance in conjunction with the vehicle velocity is used to control the spacing in front of a vehicle so that there is sufficient distance to stop in case of an emergency. The speed of the front vehicle can be measured on the following vehicle and this may be used to operate the system at still closer headways. However, in those cases where a vehicle can stop dead, as in the case of a derailment, the safety margin would be reduced.

Two subsystems are used so as to provide safety by means of redundancy. The emergency brakes on a vehicle are applied if either subsystem indicates that it is too close to the vehicle in front for safety. The emergency brakes are also applied if the headway control signals from the two subsystems do not agree within the tolerance permitted.

A problem with an unguided radar is that the vehicle is unable to see around bends in the track unless a very wide beam is used and trackside obstacles at the bend have been cleared. The energy reflected back to the transmitter will be small and expensive receiving equipment will be required. Consequently a guided radar is considered preferable.

Previous researchers have considered the extent of the range (from a few feet to a few hundred feet) to be a problem when using radar techniques in this application. However, this problem is overcome by transmitting a long pulse for the larger distances, switching to a shorter pulse as the distance falls below a certain value. The long pulse contains more power which increases the range. The short pulse does not overlap the return signal when short distances are involved.

B. The Track as a Surface Waveguide

In Sec. III, several waveguides suitable for a guided radar system were described. A major disadvantage of these systems is the cost of mounting and aligning waveguides alongside a track. Such exposed equipment is also vulnerable to damage and vandalism. To overcome these difficulties, it seems logical to attempt to use the track structure for supporting the waveguide or as part of the waveguide. The minor difficulties caused for track maintenance are not considered significant. It is necessary for the field to leak outside the waveguide so that it may contact the vehicle, while at the same time following the waveguide around the bends; therefore, a surface wave is a logical choice.^{3,16}

The problem arose of finding a method of analyzing surface waves on complicated shaped structures where dissipation was present. It was found that the finite element method in conjunction with the method of weighted residuals could be generalized for the presence of lossy materials to deal with this problem (Sec. V.) The track waveguide is considered to be a linear system causing attenuation and phase change of the input, and it is this variation of complex propagation constant with frequency which is determined by means of the numerical method. No distortion will occur if the group velocity and attenuation are constant with frequency.

The track can be expected to provide a somewhat lossy guide because the material shapes and properties are determined by factors such as tensile strength and durability rather than suitability as a waveguide. Consequently, it may be more suitable for vehicle detection in a collision avoidance system or in conjunction with a block arrangement for an Urban Transportation System, than for other communications. The track may need to be adapted to form a suitable waveguide.

An interesting problem arises when periodic discontinuities occur in the waveguide. Examples are the gaps in the case of nonwelded rails in the ties. The effect of such periodic discontinuities for wayside communications was recently examined by Hu et al.⁵ The reflections caused by the discontinuities for many sections is shown to produce a compounded effect and is not just linearly additive. The comb filter effect of the discontinuities must be considered in selecting the wave shape and carrier frequency for transmission along the guide.

Figures 3a, 3b, and 3c show a track, such as that used for the Morgantown Personal Rapid Transit and the Dallas Airport Systems, adapted for use as a surface waveguide. Figure 3d shows a rail augmented by a dielectric strip to form a waveguide. This arrangement is analyzed using the method developed in Sec. V and some of the results are presented in Sec. VI.

V. Numerical Method for Analyzing Surface Waves on Dissipative Guides

The application of the finite element method was generalized to include lossy materials. Propagation in the z direction is assumed according to $e^{-j(\omega t - \gamma z)}$ where $\gamma = \beta + j\alpha$ and ω is real. Loss is introduced through the complex dielectric constant $\epsilon = \epsilon' + j\epsilon''$, for a conductor $\epsilon'' = \sigma/\omega$. The equations following are normalized³ using a fixed unit of length which is normally a suitable dimension in the problem. For the remainder of this paper the variables are assumed to be dimensionless. These assumptions result in the transverse harmonic wave equations,

$$(\nabla^2 + k^2 - \gamma^2)E_z = 0, \quad (\nabla^2 + k^2 - \gamma^2)H_z = 0 \quad (1)$$

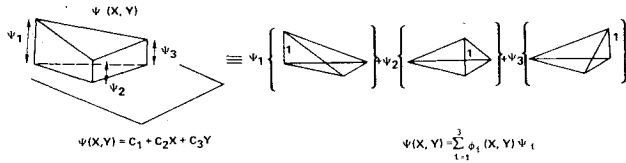


Fig. 4 Illustrated transformation from unknown polynomial coefficients c_i to unknown field values ψ_i .

where E_z and H_z are the complex electric and magnetic field components, k the intrinsic propagation constant depends on frequency ω , and ∇^2 is the transverse Laplacian operator.

Piecewise planes,

$$\psi(x, y) = c_1 + c_2 x + c_3 y \quad (2)$$

where $\psi(x, y)$ is equivalent to either E_z or H_z in Eq. (1), were chosen to approximate the functions E_z and H_z in each of the homogeneous triangular regions into which the cross section is divided. The unknown coefficients c_i are to be determined so that Eq. (2) gives in some way the best approximation to the solution of Eq. (1).

It is more advantageous to have as unknowns the values ψ_i , $i=1$ to 3 of the function $\psi(x, y)$ at the three vertices of the

$$= \omega^2 \begin{bmatrix} \epsilon'_{r1} T & -\epsilon'_{r1} T & 0 & 0 \\ \epsilon'_{rk} T & -\epsilon'_{rk} T & 0 & 0 \\ \epsilon'_{r1} T & \epsilon'_{r1} T & 0 & 0 \\ 0 & 0 & \mu'_{r1} T & -\mu'_{r1} T \\ 0 & 0 & \mu'_{rk} T & -\mu'_{rk} T \\ 0 & 0 & \mu'_{r1} T & \mu'_{r1} T \\ 0 & 0 & \mu'_{rk} T & \mu'_{rk} T \end{bmatrix} \begin{bmatrix} a'_1 \\ a'_p \\ a''_1 \\ a''_p \\ b'_1 \\ b'_p \\ b''_1 \\ b''_p \end{bmatrix} \quad (6)$$

triangle cross section. This can be achieved by transforming Eq. (2) to,

$$\psi(x, y) = \sum_{i=1}^3 \phi_i(x, y) \psi_i \quad (3)$$

as illustrated in Fig. 4, where the ϕ_i are computed algebraically.³

The approximation function $\psi(x, y)$ for the i th triangle is assumed to have a zero value in all other triangles. Therefore the approximation functions for the complete waveguide cross section may be written,

$$E_z = \sum_{i=1}^n a_i \phi_i(x, y), \quad H_z = \sum_{i=1}^n b_i \phi_i(x, y) \quad (4)$$

where a_i are the unknown complex values of the electric field E_z at the n triangle nodes in the cross section, and b_i are the unknown values for the magnetic field H_z at these nodes.

The method of weighted residuals is now applied.¹⁷ The approximation functions, Eq. (4), are substituted into Eq. (1) producing residual functions in place of zeros because only approximations for E_z and H_z were used. To set some average value of the residuals to zero the inner product is taken with $\partial E_z / \partial a_i$ and $\partial H_z / \partial b_i$ respectively and the inner product terms containing the residuals are set to zero. For $i=1$ to n ,

$$\langle \partial E_z / \partial a_i, \nabla^2 E_z \rangle + \langle \partial E_z / \partial a_i, (k^2 - \gamma^2) E_z \rangle = 0 \quad (5a)$$

$$\langle \partial H_z / \partial b_i, \nabla^2 H_z \rangle + \langle \partial H_z / \partial b_i, (k^2 - \gamma^2) H_z \rangle = 0 \quad (5b)$$

Because linear functions are used to approximate E_z and H_z the Laplacian in Eq. (5) will go to infinity at the triangle interfaces and zero elsewhere. This is not a realistic ap-

proximation to the Laplacian of the actual field, consequently the Laplacian is expanded using Green's Theorem.

Equation (1) which applies to a homogeneous region, is used in conjunction with Maxwell's equations, which apply across the interface boundaries, to produce an equation for the complete cross section composed of homogeneous triangular regions of different materials. The resulting equation can be written in the form of a matrix eigenvalue equation

$$\begin{bmatrix} C'_1 S & -C'_1 S & E'_1 W & -E'_1 W \\ C'_k S & -C'_k S & E'_k W & -E'_k W \\ C''_1 S & C''_1 S & E''_1 W & E''_1 W \\ C''_k S & C''_k S & E''_k W & E''_k W \\ -E'_1 V & E'_1 W & D'_1 S & -D'_1 S \\ -E'_k V & E'_k W & D'_k S & -D'_k S \\ -E''_1 W & -E''_1 W & D''_1 S & D''_1 S \\ -E''_k W & -E''_k W & D''_k S & D''_k S \end{bmatrix} \begin{bmatrix} a'_1 \\ a'_p \\ a''_1 \\ a''_p \\ b'_1 \\ b'_p \\ b''_1 \\ b''_p \end{bmatrix}$$

The values $C'_i, C''_i, D'_i, D''_i, E'_i, E''_i$ are for the i th triangle out of k triangles and depend on the complex propagation constant divided by angular frequency γ/ω , the complex relative permittivity ϵ_{ri} , and the complex relative permeability μ_{ri} . The coefficients a'_j, a''_j, b'_j , and b''_j are the unknown values at the $p=3k$ triangle nodes for the real part of the electric field, the imaginary part of the electric field, the real part of the magnetic field and the imaginary part of the magnetic field, respectively. S, W , and T are precomputed 3×3 matrices.¹⁸

The unsymmetric matrix eigenvalue equation is solved using a $Q-Z$ algorithm.¹⁹ In the cases where the eigenvalues are real, they represent the frequencies of possible modes of propagation and the eigenvectors determine the complex electric and magnetic fields at the triangle nodes in the cross section.

It is necessary to select initial values for the propagation constant divided by frequency $\gamma/\omega = \alpha/\omega + j\beta/\omega$. Figure 5 shows how the extrapolation of a value of propagation constant is performed corresponding to a particular frequency. For small variations from a real frequency, the imaginary part of ω^2 was generally found to vary linearly. The numerical method was tested by analyzing waveguides of simple shape for which some analytical results are available.^{3,20}

VI. Results for a Railroad Track

A. Model

An augmented rail, Figure 3d was analyzed as a surface waveguide because there are 220,000 miles of rail currently in use in the U.S., and steel rails have been chosen for two of the newest passenger systems, BART and Washington. This augmented structure supports a surface wave which is par-

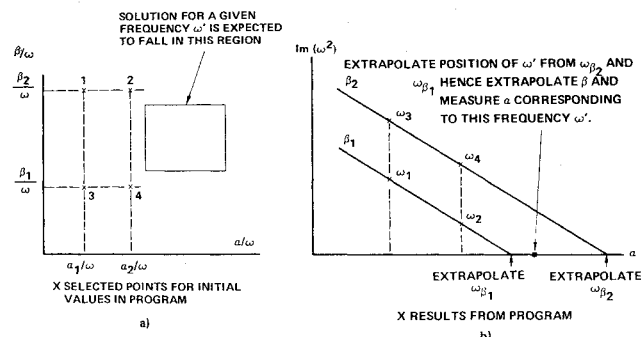


Fig. 5 a) Initial choice of β/ω and α/ω ; b) extrapolation of ω , α , and β .

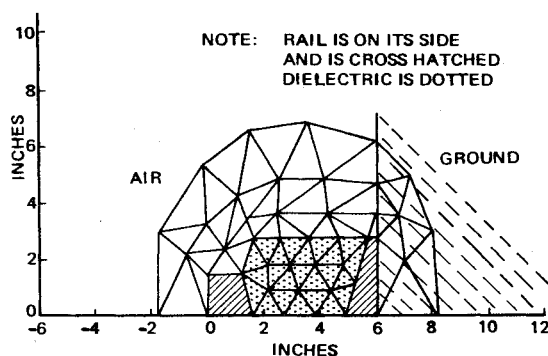


Fig. 6 Contracted layout of triangles for application of finite element method to an augmented rail.

tially trapped inside the dielectric by repeated internal reflection as the wave proceeds along the waveguide. The lowest order mode which propagates is the HE_{11} mode. It was expected that the loss in the rail would be small relative to the loss in the dielectric and the ground because of the high rail conductivity; therefore initially this loss was neglected. A perturbation method was used later and it was found that the loss due to the rail was less than 2% of that due to the ground and dielectric together.

A cross section of the rail and surroundings were subdivided into triangles (Fig. 6). The radial extent of the triangles was chosen so that a sufficient part of the field would be in the air and available for coupling with the vehicle and at the same time the field would not extend too far into the lossy ground. A larger and smaller triangle spread were used for higher and lower frequencies. Only one side of the rail was considered and symmetry about a vertical plane was assumed. The range of propagation constants was chosen to avoid too much truncation of the field at the corresponding low-frequency end of the range and poor triangle utilization at the corresponding high frequency end.

B. Results

The finite element method was applied to the augmented rail model and the results obtained for the normalized phase constant vs normalized frequency are shown in Fig. 7. Examination of the HE_{11} -type field patterns for three layouts showed that a particular layout is only valid over a small range of frequencies. The true curve is expected to lie between the curves shown. The group velocity $d\omega/d\beta$ which determines the phase distortion of the signal is seen to be approximately constant over the range of frequencies considered. Consequently, phase distortion is expected to be minimal at these frequencies.

The normalized attenuation constant divided by normalized frequency (Fig. 8) is extrapolated from the three dotted curves and agrees with the predicted behavior. At very high frequencies the wave withdraws into the dielectric material and approaches a plane wave with a

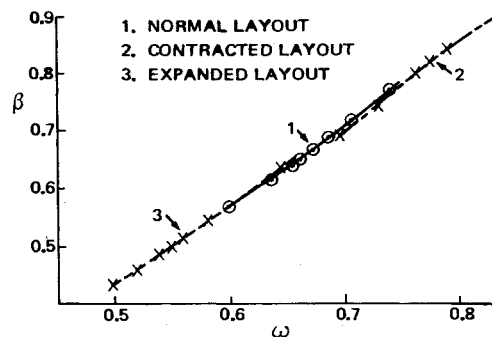


Fig. 7 Phase constant vs frequency for an augmented rail.

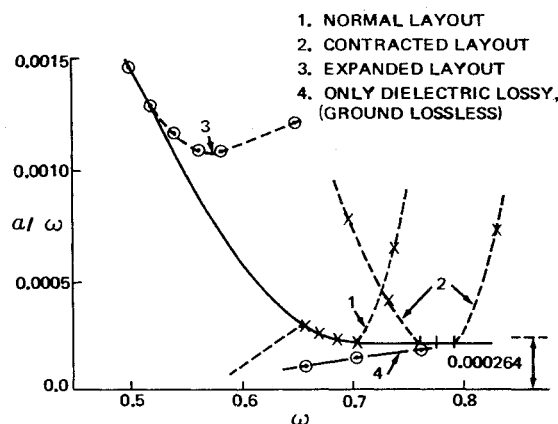


Fig. 8 Attenuation constant divided by frequency vs frequency for an augmented rail.

propagation constant $\gamma/\omega = 1.6 + j0.000264$ (Fig. 8). At lower frequencies, the wave spreads farther from the rail which simplifies coupling with the vehicle but causes an increase in attenuation (Fig. 8) because more of the field enters the ground. The predicted behavior is further confirmed by curve 4 (Fig. 8), where only the dielectric was considered lossy. Further, the attenuation for the lossy dielectric alone, and the lossy ground alone are additive.

The lowest attenuation occurs at 1.34 GHz, and at this frequency the wave will have traveled 155 meters before its amplitude has fallen by $1/e$ of the value at the transmitter. Lower attenuations may be obtained by using less lossy dielectric material.

The rail resting on ground saturated with water was investigated and the result was a lower attenuation than that for a rail resting on dry sand. The reason is evident by observing the field pattern which is now largely expelled from the ground. Detailed results including the electric and magnetic field patterns at half cycle periods for the various problems discussed are being submitted for publication.

VII. Conclusions

A guided radar was proposed as a close headway control and collision avoidance system. The reasons for suggesting a high-frequency method were explained by considering the currently used block system. The idea of guiding the radar by using the track as a waveguide was proposed after reviewing the more costly arrangements involving waveguides alongside the track. A numerical method of analyzing lossy waveguides of arbitrary shape was developed. The method was applied to an adapted rail and the results showed the feasibility of the proposed method.

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